Nowhere-zero group irregular labelings of graphs

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We investigate the group irregularity strength, \(s_g(G)\), of a graph, i.e. the least integer \(k\) such that taking any Abelian group \(G\) of order \(k\), there exists a function \(f : E(G) \rightarrow G\) so that the sums of edge labels incident with every vertex are distinct. So far the best upper bound on \(s_g(G)\) for a general graph \(G\) was exponential in \(n − c\), where \(n\) is the order of \(G\) and \(c\) denotes the number of its components. In this note we prove that \(s_g(G)\) is linear in \(n\), namely not greater than \(2n\). In fact, we prove a stronger result, as we additionally forbid the identity element of a group to be an edge label or the sum of labels around a vertex. We consider also locally irregular labelings where we require only sums of adjacent vertices to be distinct. For the corresponding graph invariant we prove the general upper bound: \(\Delta(G) + \text{col}(G) − 1\) (where \(\text{col}(G)\) is the coloring number of \(G\)) in the case when we do not use the identity element as an edge label, and a slightly worse one if we additionally forbid it as the sum of labels around a vertex. In the both cases we also provide a sharp upper bound for trees and a constant upper bound for the family of planar graphs.